Transition from Diffusive to Localized Regimes in Surface Corrugated Optical Waveguides.

A. García-Martín, J.A. Torres, J.J. Sáenz[a)]
Departamento de Física de la Materia Condensada
and Instituto de Ciencia de Materiales "Nicolás Cabrera".
Universidad Autónoma de Madrid, E-28049 Madrid, Spain.

M. Nieto-Vesperinas
Instituto de Ciencia de Materiales. Consejo Superior de Investigaciones Científicas.
Campus de Cantoblanco, E-28049 Madrid, Spain.
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Exact calculations of the transmittance of surface corrugated optical waveguides are presented. The elastic scattering of diffuse light or other electromagnetic waves from a rough surface induces a diffusive transport along the waveguide axis. As the length of the corrugated part of the waveguide increases, a transition from the *diffusive* to the *localized* regime is observed. This involves an analogy with electron conduction in nanowires, and hence, a concept analogous to that of "resistance" can be introduced. We show an oscillatory behavior of both the elastic mean free path and the localization length versus the wavelength.

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The influence of inhomogeneities and defects in the transmission properties of waveguides for both quantum and classical waves is a subject of increasing research[1–6]. The consequences of volume defects on electron wave propagation through two dimensional (2D) wires have been recently addressed[7–9]. On the other hand, some aspects of the effects of slight surface roughness and volume inhomogeneities both in optical fibers and waveguides have been under study for a long time[10,11], mainly in connection with their attenuation; however, the study of these effects has been to date concentrated on the wave scattering into the radiation field; therefore, no analysis has been made so far of the coupling between forward and backward modes[12], neither on the influence of surface roughness induced localization effects[13] into both this coupling and the radiation losses.

Although some analogies between electron waves and light and other classical waves have been put forward in apertures[2], in this letter we address localization effects of light and other electromagnetic waves propagating through a 2D corrugated waveguide (see inset of Fig. 1). There are two main differences between electron transport and the common situation in optical guides. First, the electron waves are confined within the conductor and, for small voltages, they *do not couple* with the radiation field. Second, the electron modes are mutually *incoherent*. So to resemble this picture we consider a waveguide with *perfectly conducting* walls, and the incident modes to be mutually *incoherent*. This can be achieved for example by previously passing the incident light through a moving diffuser so that correlation between phases of different modes is lost.

The corrugated part of the waveguide, of total length L, is composed of n slices of length l. The width of each slice has random values uniformly distributed between $W_0 - \delta$ and $W_0 + \delta$ about a mean value W_0. We shall take $W_0/\delta = 7$ and $l/\delta = 3/2$ (i.e. $2\delta/W_0 = 0.286$ and $l/W_0 = 0.214$). The main transport properties do not depend on the particular choice of these parameters, however. s-polarization with the electric vector parallel to the grooves (TE modes) is assumed. Transmission and reflection coefficients are exactly calculated on solving the 2D wave equation by mode matching at each slice, together with a generalized scattering-matrix technique, whose details can be found in Ref. 14.

We define the normalized transmittance of the waveguide as $G = \sum_{ij} T_{ij}$, where T_{ij} is the ratio between the total flux transmitted from the incoming mode i into the outgoing mode j, Φ_j^{out}, and the total flux of this incoming mode, Φ_i^{in}, i.e. $T_{ij} = \Phi_j^{out}/\Phi_i^{in}$, and the sum runs over the total number of propagating modes. We also define the optical analogue of the "resistance" as the inverse of the transmittance[8] $R = 1/G$. Ensemble averages, denoted by $\langle . \rangle$, are performed over 100 realizations of the corrugated waveguide.

For a perfect waveguide of constant width W_0, the normalized transmittance $\langle G \rangle$ is simply given by the number of propagating modes N, i.e. $\langle G \rangle = N = \mathrm{E}\{2W_0/\lambda\}$. A plot of $\langle G \rangle$ versus W_0/λ (see Fig.1) shows a staircase behavior which constitutes the analogous for classical waves of the conductance quantization of electronic systems[15]. Fig. 1 also contains $\langle G \rangle$ versus W_0/λ for different length values L of the corrugated portion of the waveguide. For moderate lengths L, the transmittance shows a dip just at the onset of a new propagating mode. Similar dips appear in the case of scattering by volume defects[6,16].

As the length L of the corrugated region increases, $\langle G \rangle$ presents an oscillatory behavior that, as we show below, reflects successive transitions from *diffusive* to *localized* regimes. This is done on analysing the dependence of

$\langle R\rangle$ and $\langle \ln(G)\rangle$ on L. Note that, while in clean systems the transport is *ballistic* and consists essentially of unscattered waves, (the mean free path ℓ is then much larger than both L and W_0), in a randomly corrugated waveguide the two regimes, aforementioned above, are clearly visible as shown in Fig. 2, which contains $\langle R\rangle$ and $\langle \ln(G)\rangle$ versus L, (cf. Fig. 2(a) and 2(b), respectively), for $W_0/\lambda = 2.6$. As L increases from zero, $\langle R\rangle$ follows first an almost perfect linear behavior with L which corresponds to the *semi-ballistic*[6] regime. In this case, there is so much scattering in the longitudinal direction that the transport is *diffusive* while in the transversal direction almost no scattering occurs ($W_0 < \ell < L$). Then, in analogy with Ohm's law for electron wires, the waveguide "resistance" can be described by an "ohm-like" term proportional to L/W_0 plus a "contact resistance" R_c[8,9],

$$\langle R\rangle = R_c + \rho\frac{L}{W_0} = R_c + \frac{L}{N\ell} \quad , \tag{1}$$

where the "resistivity", ρ, and the mean free path ℓ are related by[9] $\ell/W_0 = 1/(N\rho)$.

This linear behavior breaks down as L increases further. Then, a typical linear decrease of $\langle \ln(G)\rangle$ with L appears, as shown in Fig. 2(b). The system has now entered in the *localized regime* at which $W_0 < \ell \ll L$. The localization length ξ is defined by[17,18]

$$\xi \equiv -\left(\frac{\partial\langle \ln(G)\rangle}{\partial L}\right)^{-1} \quad . \tag{2}$$

The contact resistance R_c and the effective mean free path ℓ/W_0 can be easily obtained both from Eq. 1 and by a least-square fitting of the linear part in Fig. 2(a). In Fig. 3(a) we show these values of R_c and ℓ/W_0 versus W_0/λ. The contact resistance (see the inset of Fig. 3) is slightly higher than the expected value $1/N$, due to the transition region between ballistic and semi-ballistic regimes. The mean free path oscillates as W_0/λ increases, having its minima close to half-integers of W_0/λ, which correspond to the appearence of a new propagating mode in the waveguide, and thus establishes the above quoted connection with the transmittance dips at the onset of new modes shown in Fig. 1.

The dependence of the localization length ξ on the wavelength (Fig. 3(b)) can be also obtained from least-square fitting of the linear part of the plot of $\langle \ln(G)\rangle$ versus L in Fig. 2(b). It should be noted that, within the numerical accuracy of our calculation, $\xi \approx N\ell$ as shown in Fig. 3(b). This again has an analogy in agreement with the expected behavior for electron transport through disordered media[19,20].

The wavelength dependence of both the localization length and the mean free path shows that, for a fixed length, it is possible to pass on from the localized regime into the diffusive one by changing the wavelength of the incoming wave.

In conclusion, we have shown that diffuse light and other electromagnetic wave propagation through surface corrugated perfectly reflecting waveguides, constitutes the optical analogue of electron transport in disordered nanowires. As such, the concept of "averaged resistance" and the optical analogous of "Ohmic" behavior in the semi-ballistic (diffusive) regime can be introduced for these systems. We have also shown that both the effective mean free path and the localization length oscillate with the wavelength. This oscillatory transition from diffusive to localized behavior should be observed in optical waveguides. The influence of surface induced localizaton effects on radiation losses in actual waveguides remains an open question. We hope that our results will estimulate both experimental and theoretical research in this direction.

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[a] To whom correspondence should be addressed; e-mail: juanjo.saenz@uam.es.

[1] C.W.J Beenakker and H. van Houten, in *Solid State Physics: Advances in Research and Applications*, H. Ehrenreich and D. Turnbull (Academic, San Diego, 1991),Vol. 44; Ping Sheng *"Introduction to Wave Scattering, Localization and Mesoscopic Phenomena"*, Academic Press, N.Y. (1995).

[2] H. Van Houten and C.W.J. Beenaker, in *Analogies in Optics and Microelectronics*, W. Van Haeringen and D. Lenstra, eds, Kluwer, Dordrecht, 1990, pp. 203-225.

[3] R. Dalichaouch, J.P. Armstrong, S. Schultz, P.M. Platzman and L. McCall, Nature **354**, 53 (1991).

[4] H. Tamura and T. Ando, Phys. Rev. B **44**, 1792 (1991); T. Ando and H. Tamura, Phys. Rev. B **46**, 2332 (1992).

[5] K. Nikolić and A. MacKinnon, Phys. Rev. B **47**, 6555 (1993).

[6] Th.M. Nieuwenhuizen, Europhys. Lett. **24**, 269 (1993).

[7] P. García-Mochales, P.A. Serena, N. García and J.L. Costa-Krämer, Phys. Rev. B **53**,10 268 (1996).

[8] A. Mosk, Th.M. Nieuwenhuizen and C. Barnes, Phys. Rev. B **53**, 15914 (1996); Th. M. Nieuwenhuizen and J.M. Luck, Phys. Rev. E **48**, 569 (1993).

[9] T.N. Todorov, Phys. Rev. B **54**, 5801 (1996).

[10] D. Marcuse, Bell Syst. Tech. J., **48**, 3233 (1969); D. Marcuse, "Theory of Dielectric Optical Waveguides", Academic Press, N.Y. 1974.

[11] D.G. Hall, Opt. Lett. **6**, 601 (1981).

[12] F. Ladouceur and L. Polodian, Opt. Lett. **21**, 1833 (1996).

[13] A.R. McGurn and A.A. Maradudin, Phys. Rev. B **30**, 3136 (1984); Z. Tešanović and M. V. Jarić, Phys. Rev. Lett. **57**, 2760 (1986).

[14] A. Weisshaar, J. Lary, S. M. Goodnick and V.K. Tripathi. J. Appl. Phys. **70**, 355 (1991); J.A. Torres and J.J. Sáenz, (unpublished).
[15] E.A. Montie, E.C. Cosman, G. W.'t Hooft, M.B. van der Mark and C.W.J. Beenakker, Nature **350**, 594 (1991).
[16] J. Masek, P. Lipavsky and B.Kramer, J. Phys. Cond. Matt. **1**, 6395 (1989); J. Faist, P. Guéret and H. Rothuizen, Phys. Rev. B **42**, 3217 (1990); C.S. Chu and R.S. Sorbello, Phys. Rev. B bf 40, 5941 (1989).
[17] P.W. Anderson, D.J. Thouless, E. Abrahams and D.S. Fisher, Phys. Rev. B **22**, 3519 (1980). P.W. Anderson, Phys. Rev. B **23**, 4828 (1981).
[18] P.A. Lee, A.D. Stone and H. Fukuyama. Phys Rev. B **35**, 1039 (1987).
[19] D.J. Thouless, Phys. Rev. Lett. **39** 1167 (1977).
[20] J.-L. Pichard, M. Sanquer, K. Slevin and P. Debray, Phys. Rev. Lett. **65** 1812 (1990) and references therein.

FIG. 1. Averaged transmitance $\langle G \rangle$ as a function of W_0/λ for fixed values of δ/W_0 and l/W_0, ($W_0 = 7\,\delta$ and $l = (3/2)\,\delta$). Note that the case $L/W_0 = 0$ corresponds to a flat waveguide. The inset shows a schematic view of the system.

FIG. 2. (a) $\langle R \rangle$ versus the length L of the corrugated region. The straight line represents the best fitting of the linear behavior associated to the *diffusive* transport regime (see Eq.1). The results are obtained for $W_0/\lambda = 2.6$, i.e they correspond to 5 propagating modes in the waveguide. The corresponding effective mean free path is $\ell/W_0 \approx 6.3$. (b) $\langle \ln(G) \rangle$ versus L for the same case as in (a). The best fitting of the linear part of the curve is also shown. The corresponding localization length is $\xi/W_0 \approx 34.4$.

FIG. 3. (a) Effective mean free path ℓ/W_0 versus W_0/λ. The inset shows the behavior of the contact resistance R_c together with the expected value for a perfect waveguide (dotted line). (b) Localization length ξ/W_0 (filled circles) and $N\ell$ (open circles) versus W_0/λ. $\xi \approx N\ell$, within the numerical accuracy.

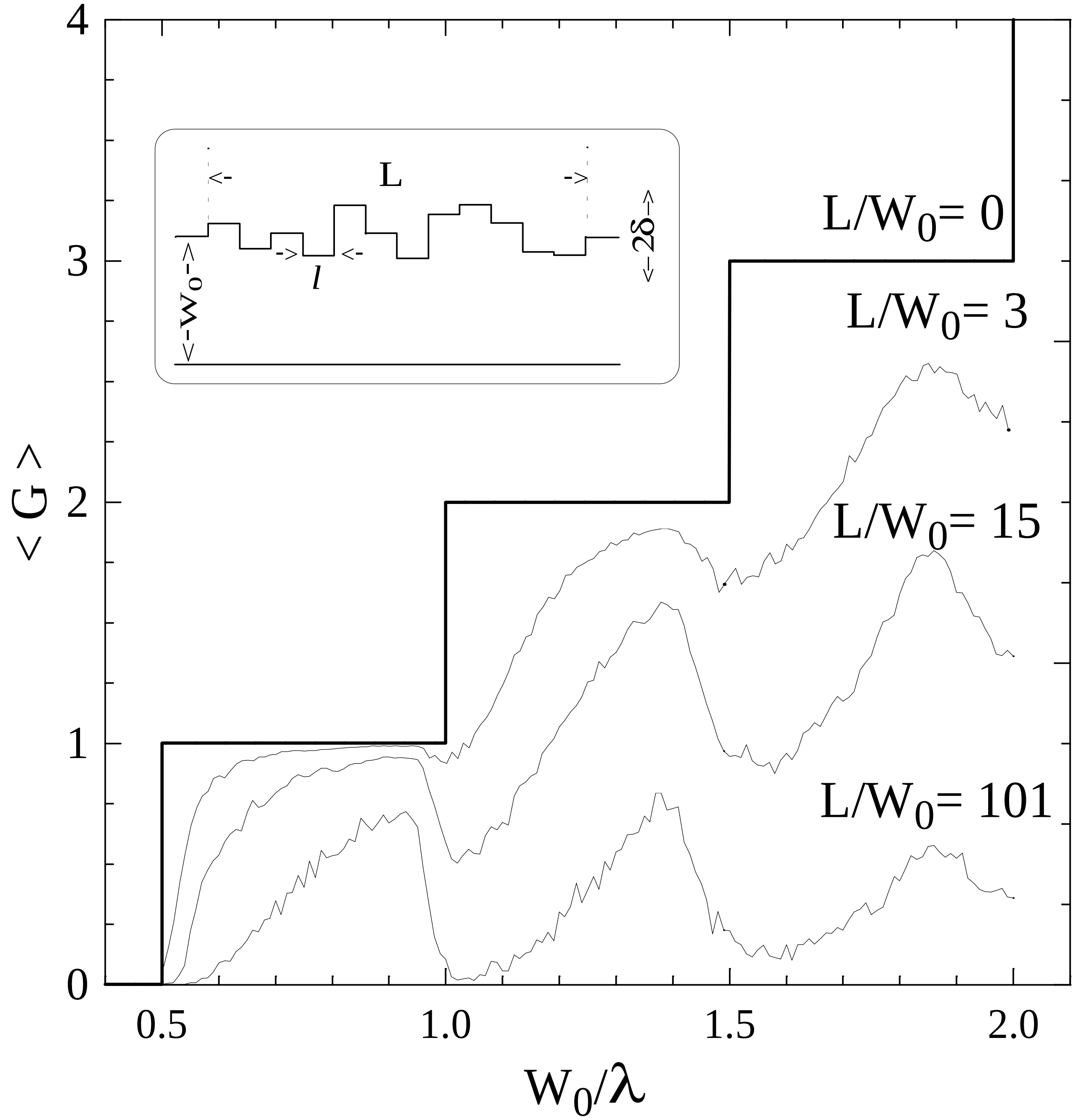

Fig. 1

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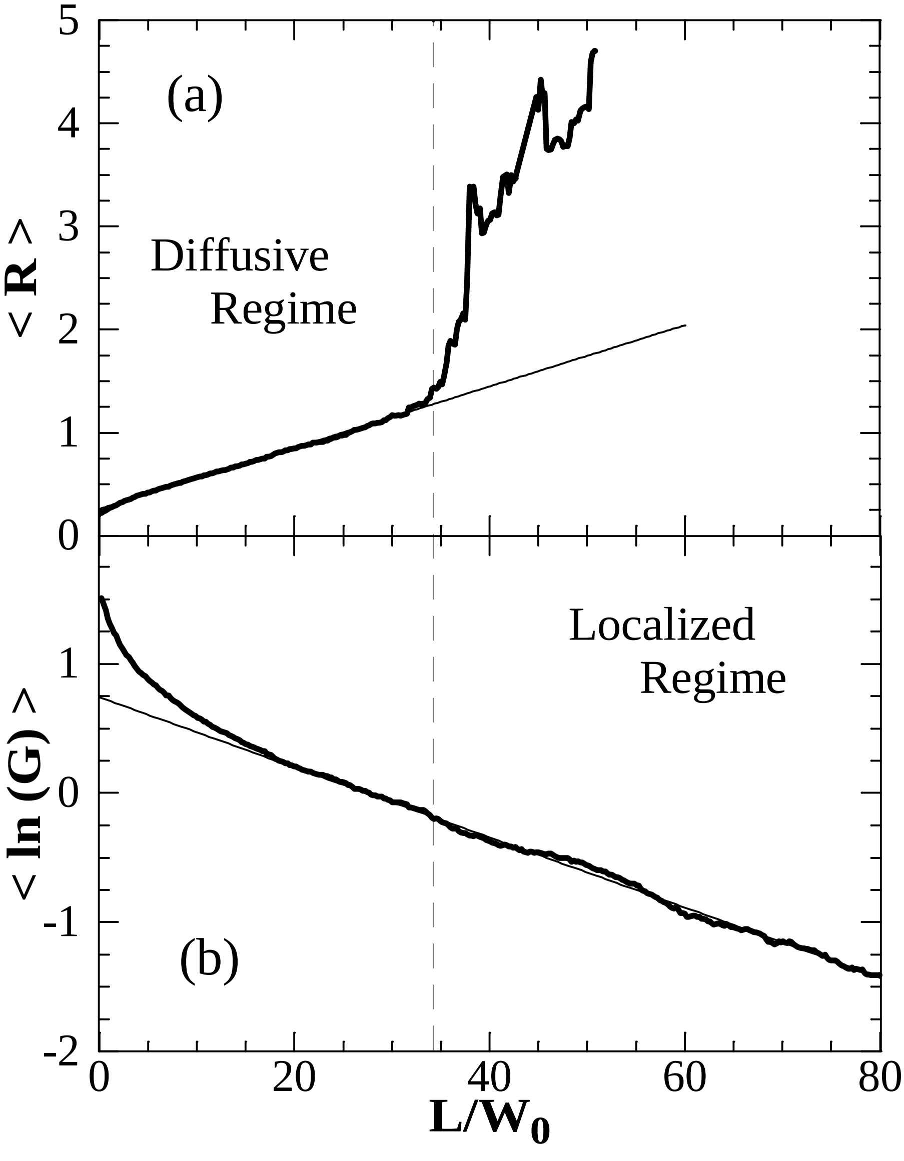

Fig. 2

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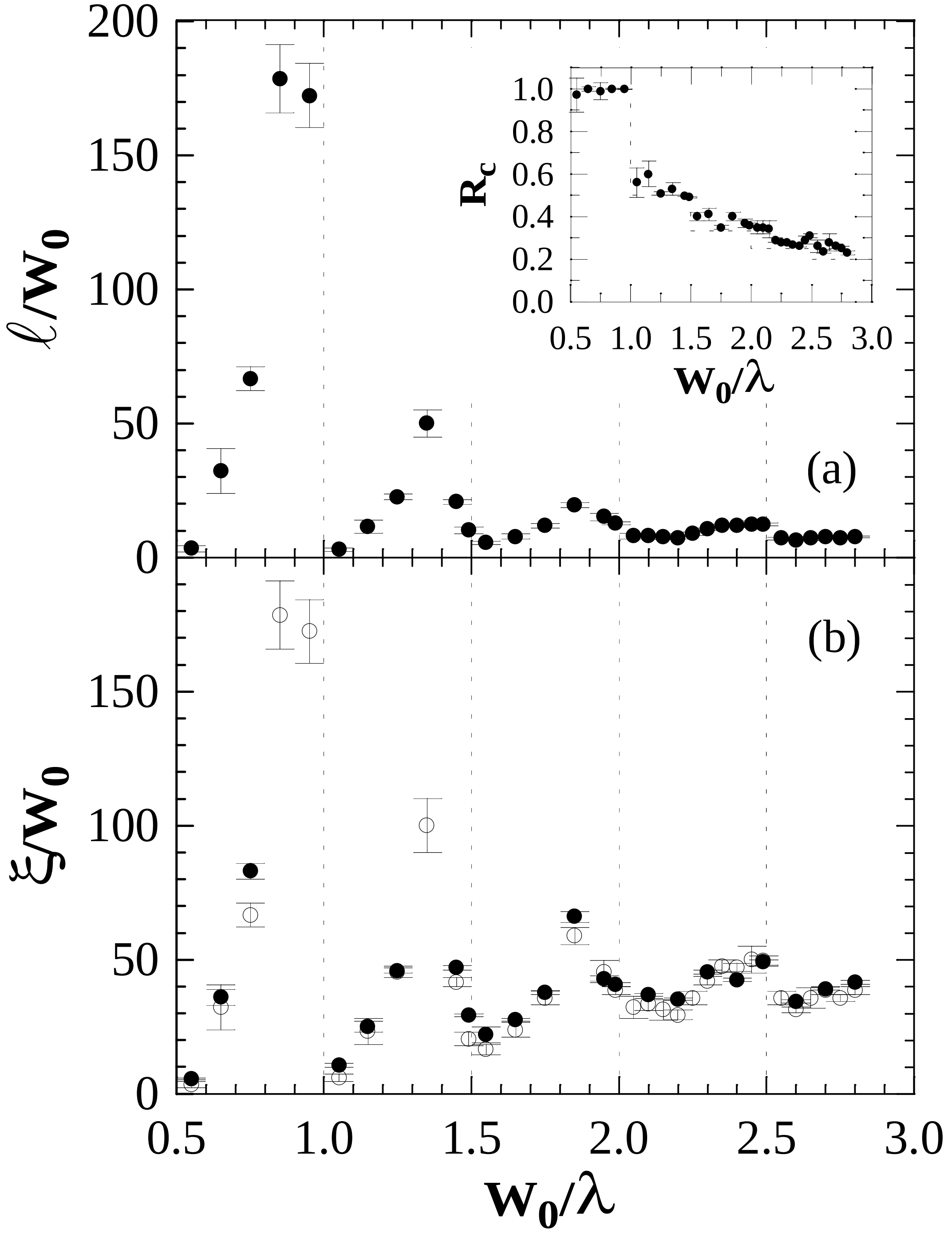

Fig. 3

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